

# Neutrino quantum states in matter

Alexander Studenikin,<sup>\*</sup> Alexei Ternov<sup>†</sup>

## Abstract

We propose a modified Dirac equation for a massive neutrino moving in the presence of the background matter. The effects of the charged and neutral-current interactions with the matter as well as the matter motion and polarization are accounted for. In the particular case of the matter with a constant density the exact solutions of this equation are found, the neutrino energy spectrum in the matter is also determined. On this basis the effects of the neutrino trapping and reflection, the neutrino-antineutrino pair annihilation and creation in a medium are studied. The quantum theory of the spin light of neutrino in matter ( $SL\nu$ ) is also developed.

## 1 Introduction

The problem of a neutrino propagation through the background matter has attracted the permanent interest for many years. The crucial importance of the matter effects was demonstrated in the studies of Refs.[1, 2] where the resonance amplification of the neutrino flavour oscillations in the presence of the matter (the Mikheyev-Smirnov-Wolfenstein effect) was discovered. The similar resonance effect in the neutrino spin oscillations in matter was considered for the first time in [3, 4].

It is a common knowledge that the matter effects in neutrino oscillations have a large impact on solar-neutrino problem (see for a recent review [5]); it may be of interest in the context of neutrino oscillation processes in supernovae and neutron stars [6]. It is also

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<sup>\*</sup>Address: Department of Theoretical Physics, Moscow State University, 119992 Moscow, Russia, e-mail: studenik@srd.sinp.msu.ru .

<sup>†</sup>Department of Theoretical Physics, Moscow Institute for Physics and Technology, 141700 Dolgoprudny, Russia, e-mail: A.Ternov@mail.ru

believed that the neutrino oscillations in the presence of matter play important role in the early Universe [7].

The neutrino interaction with matter can bring about new phenomena that do not exist in the absence of matter. In particular, we have recently shown [8, 9] that a massive neutrino moving in the background matter and electromagnetic fields can produce a new type of the electromagnetic radiation. We have named this radiation as the "spin light of neutrino" ( $SL\nu$ ) [8] in order to manifest the correspondence with the magnetic moment dependent term in the radiation of an electron moving in a magnetic field (see [10]). The radiation of a neutrino moving in a magnetic field (the  $SL\nu$  in a magnetic field) was studied before in [11]. Recently we have also considered [12] the  $SL\nu$  in gravitational fields.

In [8, 9] we develop the quasi-classical theory of the  $SL\nu$  on the basis of the quasi-classical description [13] of the neutrino spin evolution in the background matter. However, the  $SL\nu$  is a quantum process which originates from the quantum spin flip transitions. It is important to revise the calculations of the rate and total power of the  $SL\nu$  in matter using the quantum theory.

The goal of this paper is to present a reasonable step forward in the study of a neutrino interaction in the background matter and external fields. We derive a new quantum equation for the neutrino wave function, in which the effects of the neutrino-matter interaction are accounted for. This equation establishes the basis for the quantum treatment of a neutrino moving in the presence of matter. In the limit of a constant matter density, we get the exact solutions of this equation, classify them over the neutrino spin states and determine the energy spectrum. We show how the neutrino energy is disturbed by the presence of matter and also find the dependence of the energy on the neutrino helicity. From the exact expression for the neutrino energy spectrum in the background matter it follows that, for the given neutrino momentum, the energy of the negative-helicity neutrino in matter exceeds the energy of the positive-helicity neutrino. In the performed below analysis of the neutrino energy spectrum in matter we confirm some of the results on neutrinos trapping and spontaneous neutrino-antineutrino pair creation in a dense medium that have been obtained previously in [14, 17, 15, 16].

Then with the use of the obtained neutrino wave functions in matter we develop the quantum theory of the  $SL\nu$  and calculate the rate and power of the spin-light radiation in matter accounting for the emitted photons polarization. The existence of the neutrino-spin self-polarization effect [8, 9] in the matter is also confirmed within the solid base of the

developed quantum approach<sup>1</sup>.

## 2 Dirac equation for neutrino in matter

To derive the quantum equation for the neutrino wave function in the background matter we start with the effective Lagrangian that describes the neutrino interaction with particles of the background matter. For definiteness, we consider the case of the electron neutrino  $\nu$  propagating through moving and polarized matter composed of only electrons (the electron gas). The generalizations for the other flavour neutrinos and also for more complicated matter compositions are just straightforward. Assume that the neutrino interactions are described by the extended standard model supplied with  $SU(2)$ -singlet right-handed neutrino  $\nu_R$ . We also suppose that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, the interaction of a neutrino with the matter (electrons) is coherent. In this case the averaged over the matter electrons addition to the vacuum neutrino Lagrangian, accounting for the charged and neutral interactions, can be written in the form

$$\Delta L_{eff} = -f^\mu \left( \bar{\nu} \gamma_\mu \frac{1 + \gamma^5}{2} \nu \right), \quad f^\mu = \frac{G_F}{\sqrt{2}} \left( (1 + 4 \sin^2 \theta_W) j^\mu - \lambda^\mu \right), \quad (1)$$

where the electrons current  $j^\mu$  and electrons polarization  $\lambda^\mu$  are given by

$$j^\mu = (n, n\mathbf{v}), \quad (2)$$

and

$$\lambda^\mu = \left( n(\boldsymbol{\zeta} \mathbf{v}), n\boldsymbol{\zeta} \sqrt{1 - v^2} + \frac{n\mathbf{v}(\boldsymbol{\zeta} \mathbf{v})}{1 + \sqrt{1 - v^2}} \right), \quad (3)$$

$\theta_W$  is the Weinberg angle.

The Lagrangian (1) accounts for the possible effect of the matter motion and polarization. Here  $n$ ,  $\mathbf{v}$ , and  $\boldsymbol{\zeta}$  ( $0 \leq |\boldsymbol{\zeta}|^2 \leq 1$ ) denote, respectively, the number density of the background electrons, the speed of the reference frame in which the mean momenta of the electrons is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The detailed discussion on the determination of the electrons polarization can be found in [13].

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<sup>1</sup>The neutrino-spin self-polarization effect in the magnetic and gravitational fields were discussed in [11] and [12], respectively.

From the standard model Lagrangian with the extra term  $\Delta L_{eff}$  being added, we derive the following modified Dirac equation for the neutrino moving in the background matter,

$$\left\{ i\gamma_\mu \left[ \partial^\mu - \frac{1}{2}(1 + \gamma_5)f^\mu \right] - m \right\} \Psi(x) = 0. \quad (4)$$

This is the most general equation of motion of a neutrino in which the effective potential  $V_\mu = \frac{1}{2}(1 + \gamma_5)f_\mu$  accounts for both the charged and neutral-current interactions with the background matter and also for the possible effects of the matter motion and polarization. It should be noted here that the modified effective Dirac equations for a neutrino with various types of interactions with the background environment were used previously in [19, 14, 20, 21, 22, 16] for the study of the neutrino dispersion relations and derivation of the neutrino oscillation probabilities in matter. If we neglect the contribution of the neutral-current interaction and possible effects of motion and polarization of the matter then from (4) we can get corresponding equations for the left-handed and right-handed chiral components of the neutrino field derived in [15].

### 3 Neutrino wave function and energy spectrum in matter

In the further discussion below we consider the case when no electromagnetic field is present in the background. We also suppose that the matter is unpolarized,  $\lambda^\mu = 0$ . Therefore, the term describing the neutrino interaction with the matter is given by

$$f^\mu = \frac{\tilde{G}_F}{\sqrt{2}}(n, n\mathbf{v}), \quad (5)$$

where we use the notation  $\tilde{G}_F = G_F(1 + 4\sin^2\theta_W)$ .

In the rest frame of the matter the equation (4) can be written in the Hamiltonian form,

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{H}_{matt}\Psi(\mathbf{r}, t), \quad (6)$$

where

$$\hat{H}_{matt} = \hat{\boldsymbol{\alpha}}\mathbf{p} + \hat{\beta}m + \hat{V}_{matt}, \quad (7)$$

and

$$\hat{V}_{matt} = \frac{1}{2\sqrt{2}}(1 + \gamma_5)\tilde{G}_Fn, \quad (8)$$

here  $\mathbf{p}$  is the neutrino momentum. We use the Pauli-Dirac representation for the Dirac matrices  $\hat{\alpha}$  and  $\hat{\beta}$ , in which

$$\hat{\alpha} = \begin{pmatrix} 0 & \hat{\boldsymbol{\sigma}} \\ \hat{\boldsymbol{\sigma}} & 0 \end{pmatrix} = \gamma_0 \boldsymbol{\gamma}, \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \gamma_0, \quad (9)$$

where  $\hat{\boldsymbol{\sigma}} = (\sigma_1, \sigma_2, \sigma_3)$  are the Pauli matrixes.

The form of the Hamiltonian (7) implies that the operators of the momentum,  $\hat{\mathbf{p}}$ , and longitudinal polarization,  $\hat{\Sigma}\mathbf{p}/p$ , are the integrals of motion. So that, in particular, we have

$$\frac{\hat{\Sigma}\mathbf{p}}{p}\Psi(\mathbf{r}, t) = s\Psi(\mathbf{r}, t), \quad \hat{\Sigma} = \begin{pmatrix} \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \hat{\boldsymbol{\sigma}} \end{pmatrix}, \quad (10)$$

where the values  $s = \pm 1$  specify the two neutrino helicity states,  $\nu_+$  and  $\nu_-$ . In the relativistic limit the negative-helicity neutrino state is dominated by the left-handed chiral state ( $\nu_- \approx \nu_L$ ), whereas the positive-helicity state is dominated by the right-handed chiral state ( $\nu_+ \approx \nu_R$ ).

For the stationary states of the equation (4) we get

$$\Psi(\mathbf{r}, t) = e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})} u(\mathbf{p}, E_\varepsilon), \quad (11)$$

where  $u(\mathbf{p}, E_\varepsilon)$  is independent on the coordinates and time. Upon the condition that the equation (4) has a non-trivial solution, we arrive to the energy spectrum of a neutrino moving in the background matter:

$$E_\varepsilon = \varepsilon \sqrt{\mathbf{p}^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2} + \alpha m, \quad (12)$$

where we use the notation

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}. \quad (13)$$

The quantity  $\varepsilon = \pm 1$  splits the solutions into the two branches that in the limit of the vanishing matter density,  $\alpha \rightarrow 0$ , reproduce the positive and negative-frequency solutions, respectively. It is also important to note that the neutrino energy in the background matter depends on the state of the neutrino longitudinal polarization, i.e. in the relativistic case the left-handed and right-handed neutrinos with equal momenta have different energies.

The procedure, similar to one used for derivation of the solution of the Dirac equation in vacuum, can be adopted for the case of a neutrino moving in matter. We apply this

procedure to the equation (4) and arrive to the final form of the wave function of a neutrino moving in the background matter:

$$\Psi_{\varepsilon, \mathbf{p}, s}(\mathbf{r}, t) = \frac{e^{-i(E_\varepsilon t - \mathbf{p}\mathbf{r})}}{2L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ s \sqrt{1 + \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \\ s \varepsilon \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 + s \frac{p_3}{p}} \\ \varepsilon \sqrt{1 - \frac{m}{E_\varepsilon - \alpha m}} \sqrt{1 - s \frac{p_3}{p}} e^{i\delta} \end{pmatrix}, \quad (14)$$

where the energy  $E_\varepsilon$  is given by (12), and  $L$  is the normalization length. In the limit of vanishing density of matter, when  $\alpha \rightarrow 0$ , the wave function (14) transforms to the vacuum solution of the Dirac equation.

The proposed new quantum equation (4) for a neutrino in the background matter, the exact solution (14) and the obtained energy spectrum (12) establish a basis for investigations of different phenomena that can appear when neutrinos are moving in the media.

## 4 Neutrino trapping and reflection in matter

Let us now consider in some detail a neutrino energy spectrum (12) in the background matter. For the fixed magnitude of the neutrino momentum  $p$  there are the two values for the "positive sign" ( $\varepsilon = +1$ ) energies

$$E^{s=+1} = \sqrt{\mathbf{p}^2 \left(1 - \alpha \frac{m}{p}\right)^2 + m^2 + \alpha m}, \quad E^{s=-1} = \sqrt{\mathbf{p}^2 \left(1 + \alpha \frac{m}{p}\right)^2 + m^2 + \alpha m}, \quad (15)$$

that determine the positive- and negative-helicity eigenstates, respectively. The energies in Eq.(15) correspond to the particle (neutrino) solutions in the background matter. The two other values for the energy, corresponding to the negative sign  $\varepsilon = -1$ , are for the antiparticle solutions. As usual, by changing the sign of the energy, we obtain the values

$$\tilde{E}^{s=+1} = \sqrt{\mathbf{p}^2 \left(1 - \alpha \frac{m}{p}\right)^2 + m^2 - \alpha m}, \quad \tilde{E}^{s=-1} = \sqrt{\mathbf{p}^2 \left(1 + \alpha \frac{m}{p}\right)^2 + m^2 - \alpha m}, \quad (16)$$

that correspond to the positive- and negative-helicity antineutrino states in the matter. The expressions in Eqs.(15) and (16) would reproduce the neutrino dispersion relations of [15] (see also [16]), if the contribution of the neutral-current interaction to the neutrino potential were left out.

The neutrino dispersion relations in the matter exhibits a very fascinating feature (see also [15, 16]). As it follows from (15) and (16), the neutrino energy may has a minimum at a non-zero momentum. It may also happen that the neutrino group and phase velocities are pointing in the opposite directions.

The obtained neutrino and antineutrino energy spectra, Eqs.(15) and (16), enable us to consider several important phenomena of a neutrino propagation in the background medium. To illustrate the key features of the obtained dispersion relations, we plot in Figs.1 and 2 the allowed ranges for the neutrino and antineutrino energies for different values of the matter density parameter  $\alpha$ . In the left side of Fig.1 the minimal energies

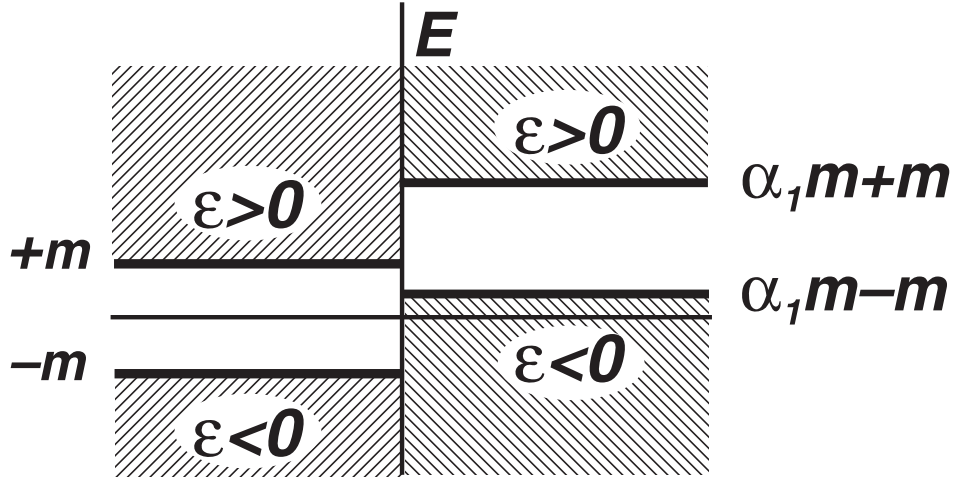


Figure 1: The ranges of the "positive" ( $\varepsilon = +1$ ) and "negative" ( $\varepsilon = -1$ ) sign energies in the vacuum (the left side) and in the presence of the matter with the density parameter  $1 < \alpha_1 < 2$  (the right side). The dashed and un-dashed zones are for the allowed and forbidden values of the energy, respectively.

for a neutrino and antineutrino in vacuum ( $\alpha = 0$ ) are shown by the solid lines given by  $E = \pm m$ . The un-dashed gap between  $E = +m$  and  $E = -m$  is the forbidden energy zone. In the right of Fig.1 the corresponding minimal values of the neutrino and antineutrino energies in the presence of the matter with the density parameter  $1 < \alpha_1 < 2$  are shown. The forbidden energy zone is lifted up by the value of  $\alpha_1 m$  with respect to the vacuum case. The existence of the two interesting phenomena can be recognized from this figure. First of all, antineutrinos with energies in the range of  $|\alpha_1 m - m| \leq E < m$  can not escape

from the medium because this particular range of energies exactly falls on the forbidden energy zone in the vacuum. In this case an antineutrino has not enough energy to survive in the vacuum, therefore it is trapped inside the medium. It should be also noted here that the possibility for a neutrino to have the minimal energy in the matter less than the neutrino mass and the corresponding neutrino trapping effect in the medium has been discussed in [16].

The second fascinating phenomenon can appear when a neutrino is propagating in the vacuum towards the interface between the vacuum and the matter. We again examine the neutrino dispersion relations illustrated by Fig.1. If the neutrino energy in the vacuum is less than the neutrino minimal energy in the medium (this case corresponds to the neutrino energies in the vacuum in the range given by  $m \leq E < \alpha_1 m + m$ ) then the neutrino will be reflected from the interface because the appropriate energy level inside the medium is not accessible for the neutrino.

## 5 Neutrino-antineutrino pair annihilation and creation in matter

Let us now consider the corresponding phenomena that can appear at the interface between the vacuum and the medium in the case when the matter density parameter  $\alpha_2 \geq 2$  (see Fig.2). Consider a neutrino with the energy  $m < E \leq \alpha_2 m - m$  propagating in the vacuum towards the interface with the matter. Let us also suppose that not all of "negative sign" energy levels in the matter are occupied and, in particular, the level with the energy exactly equal to the energy of the neutrino which is falling on the interface from the vacuum. This means that there is also the antineutrino in the matter. In this case the process of the neutrino-antineutrino annihilation can proceed at the interface of the vacuum and the matter.

From the Fig.2 that illustrates the neutrino and antineutrino dispersion relations in the vacuum and matter with the density parameter  $\alpha_2 \geq 2$  it can be also seen that the effect of the spontaneous neutrino-antineutrino pair creation can appear in the presence of the matter. Indeed, the "negative sign" energy levels in the matter (the right-hand side of Fig.2) have their counterparts in the "positive sign" energy levels in the vacuum (the left-hand side of Fig.2). The neutrino-antineutrino pair creation can be interpreted

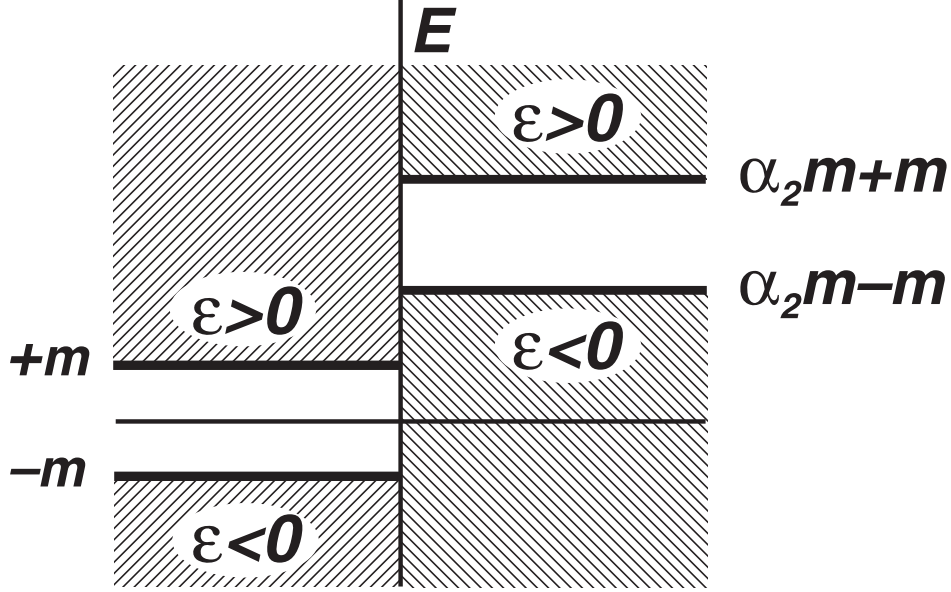


Figure 2: The ranges of the "positive" ( $\varepsilon = +1$ ) and "negative" ( $\varepsilon = -1$ ) sign energies in the vacuum (the left side) and in the presence of the matter with the density parameter  $\alpha_2 \geq 2$  (the right side). The dashed and un-dashed zones are for the allowed and forbidden values of the energy, respectively.

as the process of the appearance of the particle state in the "positive sign" energy range accompanied by the appearance of the hole state in the "negative sign" energy sea. The phenomenon of the neutrino-antineutrino pair creation in the presence of the matter is similar to the spontaneous electron-positron pair creation according to Klein's paradox of the electrodynamics (see, for instance, [23]). The possibility of the neutrino-antineutrino pair creation in the medium was also discussed before in [16, 17].

## 6 Quantum theory of spin light of neutrino in matter

In this section we should like to use the obtained solutions (14) of the equation (4) for a neutrino moving in the background matter for the study of the spin light of neutrino ( $SL\nu$ ) in the matter. We develop below the *quantum* theory of this effect. Within the quantum approach, the corresponding Feynman diagram of the  $SL\nu$  in the matter is the

standard one-photon emission diagram with the initial and final neutrino states described by the "broad lines" that account for the neutrino interaction with the matter. It follows from the usual neutrino magnetic moment interaction with the quantized photon field, that the amplitude of the transition from the neutrino initial state  $\psi_i$  to the final state  $\psi_f$ , accompanied by the emission of a photon with a momentum  $k^\mu = (\omega, \mathbf{k})$  and polarization  $\mathbf{e}^*$ , can be written in the form

$$S_{fi} = -\mu\sqrt{4\pi} \int d^4x \bar{\psi}_f(x)(\hat{\Gamma}\mathbf{e}^*) \frac{e^{ikx}}{\sqrt{2\omega L^3}} \psi_i(x), \quad (17)$$

where  $\mu$  is the neutrino magnetic moment,  $\psi_i$  and  $\psi_f$  are the corresponding exact solutions of the equation (4) given by (14), and

$$\hat{\Gamma} = i\omega \{ [\boldsymbol{\Sigma} \times \boldsymbol{\varkappa}] + i\gamma^5 \boldsymbol{\Sigma} \}. \quad (18)$$

Here  $\boldsymbol{\varkappa} = \mathbf{k}/\omega$  is the unit vector pointing the direction of the emitted photon propagation. The integration in (19) with respect to time yields

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}} 2\pi\delta(E_f - E_i + \omega) \int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\Gamma}\mathbf{e}^*) e^{i\mathbf{k}\mathbf{r}} \psi_i(\mathbf{r}), \quad (19)$$

where the delta-function stands for the energy conservation. Performing the integrations over the spatial co-ordinates, we can recover the delta-functions for the three components of the momentum. Finally, we get the law of the energy-momentum conservation for the considered process,

$$E_i = E_f + \omega, \quad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}. \quad (20)$$

Let us suppose that the weak interaction of the neutrino with the electrons of the background is indeed weak. In this case, we can expand the energy (12) over  $\tilde{G}_F n/p \ll 1$  and in the liner approximation get

$$E \approx E_0 - sm\alpha \frac{p}{E_0} + \alpha m, \quad (21)$$

where  $E_0 = \sqrt{p^2 + m^2}$ . Then from the law of the energy conservation (20) we get for the energy of the emitted photon

$$\omega = E_{i_0} - E_{f_0} + \Delta, \quad \Delta = \alpha m \frac{p}{E_0} (s_f - s_i), \quad (22)$$

where the indexes  $i$  and  $f$  label the corresponding quantities for the neutrino in the initial and final states. From Eq.(22) and the law of the momentum conservation, in the linear approximation over  $n$ , we obtain

$$\omega = (s_f - s_i)\alpha m \frac{\beta}{1 - \beta \cos \theta}, \quad (23)$$

where  $\theta$  is the angle between  $\boldsymbol{x}$  and the direction of neutrino speed  $\boldsymbol{\beta}$ .

From the above consideration it follows that the only possibility for the  $SL\nu$  to appear is provided in the case when the neutrino initial and final states are characterized by  $s_i = -1$  and  $s_f = +1$ , respectively. Thus, on the basis of the quantum treatment of the  $SL\nu$  in the matter we conclude, that in this process the left-handed neutrino is converted to the right-handed neutrino (see also [8]) and the emitted photon energy is given by

$$\omega = \frac{1}{\sqrt{2}}\tilde{G}_F n \frac{\beta}{1 - \beta \cos \theta}. \quad (24)$$

Note that the photon energy depends on the angle  $\theta$  and also on the value of the neutrino speed  $\beta$ . In the case of  $\beta \approx 1$  and  $\theta \approx 0$  we confirm the estimation for the emitted photon energy given in [8].

If further we consider the case of the neutrino moving along the OZ-axes, we can rewrite the solution (14) for the neutrino states with  $s = -1$  and  $s = +1$  in the following forms

$$\Psi_{\mathbf{p},s=-1}(\mathbf{r}, t) = \frac{e^{-i(Et-\mathbf{p}\mathbf{r})}}{\sqrt{2}L^{\frac{3}{2}}} \begin{pmatrix} 0 \\ -\sqrt{1 + \frac{m}{E-\alpha m}} \\ 0 \\ \sqrt{1 - \frac{m}{E-\alpha m}} \end{pmatrix}, \quad (25)$$

and

$$\Psi_{\mathbf{p},s=+1}(\mathbf{r}, t) = \frac{e^{-i(Et-\mathbf{p}\mathbf{r})}}{\sqrt{2}L^{\frac{3}{2}}} \begin{pmatrix} \sqrt{1 + \frac{m}{E-\alpha m}} \\ 0 \\ \sqrt{1 - \frac{m}{E-\alpha m}} \\ 0 \end{pmatrix}. \quad (26)$$

We now put these wave functions into Eq.(19) and calculate the spin light transition rate in the linear approximation of the expansion over the parameter  $\tilde{G}_F n/p$ . Finally, for the rate we get

$$\Gamma_{SL} = \frac{1}{2\sqrt{2}}\tilde{G}_F^3 \mu^2 n^3 \beta^3 \int \frac{S \sin \theta}{(1 - \beta \cos \theta)^4} d\theta. \quad (27)$$

where

$$S = (\cos \theta - \beta)^2 + (1 - \beta \cos \theta)^2. \quad (28)$$

Performing the integrations in Eq.(27) over the angle  $\theta$ , we obtain for the rate

$$\Gamma_{SL} = \frac{2\sqrt{2}}{3} \mu^2 \tilde{G}_F^3 n^3 \beta^3 \gamma^2. \quad (29)$$

This result exceeds the value of the neutrino spin light rate derived in [8] by a factor of two because here the neutrinos in the initial state are totally left-hand polarized, whereas the case of the unpolarized neutrinos in the initial state was consider in [8].

The corresponding expression for the radiation power is

$$I_{SL} = \frac{1}{4} \mu^2 \tilde{G}_F^4 n^4 \beta^4 \int \frac{S \sin \theta}{(1 - \beta \cos \theta)^5} d\theta. \quad (30)$$

Performing the integration, we get for the total radiation power

$$I_{SL} = \frac{2}{3} \mu^2 \tilde{G}_F^4 n^4 \beta^4 \gamma^4. \quad (31)$$

In the performed above quantum treatment of the  $SL\nu$  in the background matter we confirm the main properties [8, 9, 12] of this radiation. In particular, as it follows from (30), the  $SL\nu$  is strongly beamed along the propagation of the relativistic neutrino. The total power of the  $SL\nu$  in the matter is increasing with the increase of the background matter density and the neutrino  $\gamma$  factor,  $I_{SL} \sim n^4 \gamma^4$ . From the obtained within the quantum treatment expression (24) one can get an estimation for the emitted photon energy

$$\omega = 2.37 \times 10^{-7} \left( \frac{n}{10^{30} \text{cm}^{-3}} \right) \left( \frac{E}{m_\nu} \right)^2 \text{eV}. \quad (32)$$

It follows that for the matter densities and neutrino energies, appropriate for the neutron star environments, the range of the radiated photons energies may span up to the gamma-rays.

Using the neutrino transition amplitude in the matter, Eq.(19), it is also possible [24] to derive the  $SL\nu$  rate and power with the emitted photons polarizations being accounted for. Information on the photons polarization may be important for the experimental observation of the  $SL\nu$  from different astrophysical and cosmology objects and media.

Finally, the developed above quantum theory of the neutrino motion in the background matter also reveals the nature of the  $SL\nu$ . In particular, the application of the quantum

theory to this phenomenon enables us to demonstrate that the  $SL\nu$  appears due to the two subdivided phenomena: (i) the shift of the neutrino energy levels in the presence of the background matter, which is different for the two opposite neutrino helicity states, (ii) the radiation of the  $SL\nu$  photon in the process of the neutrino transition from the "exited" negative-helicity state to the low-lying positive-helicity neutrino state in matter. Therefore, as it has been discussed in [8, 9], the relativistic neutrino beam composed of the active neutrinos  $\nu_L$  can be converted to the sterile neutrinos  $\nu_R$ .

## 7 Summary

The quantum approach to description of a neutrino moving in the background matter based on the modified Dirac equation has been proposed. The exact solutions of this equation and the neutrino energy spectrum have been derived in the case of a matter with constant density. The neutrino trapping and reflection, and also the neutrino-antineutrino pair annihilation and creation in a medium have been studied. The quantum theory of the spin light of a neutrino in the matter has been also developed. The quantum method for including a matter background would have a large impact on the studies of different processes with neutrinos propagating in the astrophysical and cosmology media.

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